SNUTP-94/06 (Revised)
April, 1994

Pentaquark Exotic Baryons in the Skyrme Model

Yongseok OH

Department of Physics, National Taiwan University
Taipei, Taiwan 10764, R.O.C.

and

Byung-Yoon PARK

Department of Physics, Chungnam National University
Deajeon 305–764, Korea

and

Dong-Pil MIN

Center for Theoretical Physics and Department of Physics
Seoul National University, Seoul 151–742, Korea

ABSTRACT

We investigate the pentaquark(P) exotic baryons as soliton-antiflavored heavy mesons bound states in the limit of infinitely heavy meson mass. Our approach respects the chiral symmetry as well as the heavy quark symmetry. The results reveal a possibility for the loosely bound non-strange P-baryon(s).

A pentaquark (P) baryon [1, 2] is an exotic baryon predicted in quark models, which consists of a heavy antiquark $\bar{Q}(\bar{c}$ or $\bar{b})$ and four light quarks such as ordinary $q_0(u,d)$ quarks and s-quark. Its stability is provided by a gain in the hyperfine interaction energy stemming from one gluon exchange just as in the H-dibaryon [3]. Lipkin [1] and Gignoux *et. al.* [2] show that, in the limit of the infinite c-quark mass and the exact $SU(3)_F$ symmetry for the light quarks, a strange anti-charmed baryon $P_{\bar{c}s}$ $(\bar{c}sq_0q_0q_0)$ is stable against the decays into ΛD or ND_s. The binding energy of $P_{\bar{c}s}$ over the q_0q_0s-$\bar{Q}q_0$ or $q_0q_0q_0$-$\bar{Q}s$ system is $\sim$ 150 MeV, which becomes down to $\sim$ 85 MeV if included a realistic $SU(3)_F$ symmetry breaking [4]. Furthermore, the system becomes unbound if quark motions are taken into account [5, 6].

In the Skyrme model, the P-baryon can be described by a soliton-antiflavored heavy meson bound state (if any). Recently, Riska and Scoccola (RS) [7] reported a few *non-strange* P-baryon states such as $P_{\bar{c}}(\bar{c}q_0q_0q_0q_0)$ and $P_{\bar{b}}(\bar{b}q_0q_0q_0q_0)$ in the extended bound state approach [8] of the Skyrme model. The lowest $P_{\bar{c}}$ and $P_{\bar{b}}$ are the isosinglet (i=0) and spin doublet (j=$\frac{1}{2}$) states, whose binding energies with respect to the ND and NB threshold are estimated to be 110 $\sim$ 190 MeV and 0.7 $\sim$ 1.0 GeV, respectively. It is quite a remarkable result compared with the quark model by which a nonstrange anti-charmed P-baryon has no sufficient symmetry to be stable via the hyperfine interactions.

Following the traditional collective coordinate quantization procedure [8], RS obtained the mass formula for such nonstrange P-baryon, m_P, as

$$m_P = M_{sol} + \omega + \frac{1}{2\mathcal{I}}\{cj(j+1) + (1-c)i(i+1) + c(c-1)k(k+1)\}. \tag{1}$$

Here, M_{sol} and $\mathcal{I}$ are the soliton mass and its moment of inertia with respect to the collective isospin rotation, ω and k are the eigenenergy and the grand spin of the bound state for the antiflavored heavy meson, and $i(j)$ are the isospin(spin) of the P-baryon. It has been understood that the "hyperfine constant", c, should vanish in the heavy meson mass limit so that the masses do not depend on the spin, while their numerically obtained hyperfine constants for the bound B=+1 heavy mesons do not vanish. The worse is that they can be negative, which implies that the state with higher spin has the lower mass. On the other hand, the heavy quark spin symmetry does not necessarily lead to such an independence of mass on the spin. It just implies that the hadrons containing a single heavy quark (or a single antiquark) come in degenerate doublets of total spin j=$j_\ell \pm \frac{1}{2}$ with j_ℓ being the spin of the light degrees of freedom, while the traditional mass formula (1) is not convenient to see this kind of symmetry explicitly. Consequently, the work of RS does not respect the heavy quark symmetry. [9]

In addition, their way of treating the heavy vector mesons is not consistent with the heavy quark symmetry, either. [10] They integrate out the heavy vector meson fields in favor of the pseudoscalar ones, which may be guaranteed only when the former are sufficiently heavier than the latter. Since the vector mesons D^* and B^* are only a few percent heavier than their pseudoscalar partners D and B, one should treat the heavy vector mesons on the same footing as the heavy pseudoscalar mesons. We are to re-examine the existence of nonstrange P-baryons in the Skyrme model by correcting these defects of the work of ref. [7]. In this paper, as a first step, we report briefly our estimation on the binding energy in the infinitely heavy meson mass limit with a theory respecting the heavy quark symmetry and the chiral symmetry.

Consider a system of Goldstone pions (π^+, π^0 and π^-) and j^π=0^- and 1^- heavy mesons containing a sufficiently heavy antiquark $\bar{Q}$ and a light quark q. The dynamics of the system is governed by the $SU(2)_L \times SU(2)_R$ chiral symmetry and the heavy quark symmetry [9]. As the mass of the heavy constituent becomes sufficiently larger than the typical scale of strong

interactions, its spin decouples to the rest so that the 0^- and 1^- heavy mesons have the same mass and furthermore the dynamics of the system is independent of the heavy constituent. [9] The incorporation of the heavy quark symmetry is facilitated by representing the 0^- and 1^- heavy mesons by a 4×4 (isodoublet) matrix field $H(x)$ as

$$H = \frac{1 - \not{v}}{2} \left(\Phi_v \gamma_5 - \Phi^*_{v\mu} \gamma^\mu \right). \tag{2}$$

Here, γ's are the 4×4 Dirac matrices and $\not{v}$ denotes $v_\mu \gamma^\mu$. And Φ_v and $\Phi^*_{v\mu}$, respectively, represent the heavy pseudoscalar field and heavy vector fields moving with a four velocity v_μ. As inferred from the $q\bar{Q}$ structure, under the heavy quark spin rotation, H transforms

$$H \to H S^{-1}, \tag{3}$$

with $S \in SU(2)_v$ (the heavy quark spin symmetry group boosted by the velocity v).

As for the system of Goldstone pions, the chiral symmetry is realized in a nonlinear way by a 2×2 unitary matrix

$$\Sigma = \exp\left(\frac{i}{f_\pi} \vec{\tau} \cdot \vec{\pi} \right), \tag{4}$$

which transforms, under an $SU(2)_L \times SU(2)_R$ transformation,

$$\Sigma \to L \Sigma R^\dagger, \tag{5}$$

with global transformations $L \in SU(2)_L$ and $R \in SU(2)_R$. Here, f_π is the pion decay constant. In terms of Σ, the interactions among the Goldstone bosons are described by the lagrangian density

$$\mathcal{L}_M = \frac{f_\pi^2}{4} \,\mathrm{Tr}(\partial_\mu \Sigma^\dagger \partial^\mu \Sigma) + \cdots, \tag{6}$$

where terms with higher derivatives are denoted by the ellipsis. With a suitable stabilizing term provided, the nonlinear lagrangian $\mathcal{L}_M$ supports a classical soliton solution

$$\Sigma_0(\vec{r}) = \exp(i \vec{\tau} \cdot \hat{r} F(r)), \tag{7}$$

with the profile function satisfying the boundary conditions $F(0) = \pi$ and $F(r) \stackrel{r \to \infty}{\longrightarrow} 0$. To this purpose, among the higher derivative terms, we include the conventional Skyrme term

$$\mathcal{L}_{\mathrm{SK}} = \frac{1}{32 e^2} \,\mathrm{Tr}[\Sigma^\dagger \partial_\mu \Sigma, \Sigma^\dagger \partial_\nu \Sigma]^2, \tag{8}$$

into $\mathcal{L}_M$ with a dimensionless parameter e.

Let $\xi \equiv \Sigma^{1/2}$ be a redefined matrix which transforms under an $SU(2)_L \times SU(2)_R$ as

$$\xi \to L \xi U^\dagger = U \xi R^\dagger, \tag{9}$$

with a special unitary matrix U depending on L, R and the Goldstone fields. And let's assign $H(x)$ a transformation rule as

$$H \to U H. \tag{10}$$

Then, to the leading order in the derivatives on the Goldstone boson fields, a *"heavy-quark-symmetric"* and *"chirally-invariant"* lagrangian can be written as [11]

$$\mathcal{L}_{HQS} = \mathcal{L}_M - i v_\mu \,\mathrm{Tr}(\bar{H}(\partial^\mu + V^\mu) H) + g \,\mathrm{Tr}(\bar{H} \gamma^\mu \gamma_5 A_\mu H), \tag{11}$$

with a universal coupling constant g for the $\Phi^*\Phi\pi$ and $\Phi^*\Phi^*\pi$ interactions. Vector fields $V_\mu(x)$ and axial vector fields $A_\mu(x)$ are defined and transform under the chiral transformation as

$$\begin{aligned} V_\mu &= \tfrac{1}{2}(\xi^\dagger\partial_\mu\xi + \xi\partial_\mu\xi^\dagger) \to UV_\mu U^\dagger + U\partial_\mu U^\dagger, \\ A_\mu &= \tfrac{i}{2}(\xi^\dagger\partial_\mu\xi - \xi\partial_\mu\xi^\dagger) \to UA_\mu U^\dagger. \end{aligned} \tag{12}$$

In our model lagrangian (11), we have three parameters; the pion decay constant f_π, the Skyrme parameter e and the pion-heavy meson coupling constant g. Empirical value of f_π is 93 MeV. The nonrelativistic quark model provides a naive estimation for the value of g as $g = -\frac{3}{4}$ [12] and, in case of Q=c, the experimental upper limit [13] of the D^* width ($\sim$ 131 keV) implies $|g|^2 \lesssim 0.5$ when combined with the $D^{*+} \to D^+\pi^0$ and $D^{*+} \to D^0\pi^+$ branching ratios [14]. We will take them however as free parameters and adjust them to produce experimentally observed heavy baryon masses.

Our main interest is the heavy meson bound states to the static potentials provided by the baryon-number-one soliton configuration (7); which are explicitly

$$\begin{aligned} V^\mu &= (V^0, \vec{V}) = (0, iv(r)\hat{r}\times\vec{\tau}), \\ A^\mu &= (A^0, \vec{A}) = (0, \tfrac{1}{2}(a_1(r)\vec{\tau} + a_2(r)\hat{r}\vec{\tau}\cdot\hat{r})), \end{aligned} \tag{13}$$

with

$$v(r) = \frac{\sin^2(F/2)}{r}, \quad a_1(r) = \frac{\sin F}{r} \quad \text{and} \quad a_2(r) = F' - \frac{\sin F}{r}. \tag{14}$$

In the rest frame where $v^\mu = (1, \vec{0})$, the equation of motion for the eigenmodes $H_n(\vec{r})$ of the H-field with the eigenvalue ε_n can be read off as

$$\varepsilon_n H_n(\vec{r}) = -g\vec{\sigma}\cdot\vec{A}H_n(\vec{r}), \tag{15}$$

where we have used the relation $\vec{\gamma}\gamma_5 H(x) = \vec{\sigma}H(x)$, and n denotes a set of quantum numbers to classify the eigenmodes. The "hedgehog" configuration (7) correlates the isospin and the angular momentum, while the heavy-quark symmetry implies the heavy quark spin decoupling. Thus, the equation of motion is invariant under the parity operation, under the heavy quark spin rotation and under the simultaneous rotations in the isospin space, *light-quark* spin space and ordinary spaces. Let $\vec{L}$, $\vec{S}_\ell$, $\vec{S}_Q$ and $\vec{I}_h$ be the orbital angular momentum, light quark spin, heavy quark spin and isospin operators of the heavy mesons, respectively. And $Y_{\ell m}(\hat{r})$, $|\pm\frac{1}{2})_\ell$, ${}_Q(\pm\frac{1}{2}|$, $\tilde{\phi}_{\pm\frac{1}{2}}$ be their corresponding eigenfunctions or eigenstates, respectively. The simultaneous rotations mentioned above are generated by the "*light-quark grand spin*" operator defined as[#1]

$$\vec{K}_\ell = \vec{L} + \vec{S}_\ell + \vec{I}_h. \tag{16}$$

Then, the eigenmodes of the heavy meson can be classified by the third component of the heavy quark spin s_Q, the grand spin and its third component (k_ℓ, k_3) and the parity π. The set of quantum numbers will be denoted by $n = \{k_\ell, k_3, \pi, s_Q\}$.

The situation is very similar to obtaining the eigenmodes of the confined quarks in the chiral bag model [15]. We start with the construction of the eigenfunctions of the grand spin and the

[#1] By the subscript ℓ, we distinguish $\vec{K}_\ell$ from the traditional grand spin operator used in the bound state approach in the Skyrme model; *i.e.*, $\vec{K}(= \vec{L} + \vec{S} + \vec{I}_h)$ with $\vec{S}(= \vec{S}_\ell + \vec{S}_Q)$ being the spin operator of the heavy mesons.

Table 1 : Four $\mathcal{K}^{(i)}_{k_\ell k_3 s_Q}$-basis.

i	λ	k_ℓ
1	$\ell+\frac{1}{2}$	$\lambda-\frac{1}{2}=\ell$
2	$\ell-\frac{1}{2}$	$\lambda+\frac{1}{2}=\ell$
3	$\ell-\frac{1}{2}$	$\lambda-\frac{1}{2}=\ell-1$
4	$\ell+\frac{1}{2}$	$\lambda+\frac{1}{2}=\ell+1$

heavy quark spin by combining the direct products of the four angular momentum eigenstates, $Y_{\ell m_\ell}$, $\tilde{\phi}_{\pm\frac{1}{2}}$, $|\pm\frac{1}{2})_\ell$ and ${}_Q(\pm\frac{1}{2}|$: [16]

$$\mathcal{K}^{(i)}_{k_\ell k_3 s_Q} = \sum_{m_s,m_t} (\ell^{(i)}, m_\ell, \tfrac{1}{2}, m_t|\lambda^{(i)}, m_\ell+m_t)(\lambda^{(i)}, m_\ell+m_t, \tfrac{1}{2}, m_s|k_\ell, k_3)\; Y_{\ell m_\ell}(\hat{r})\tilde{\phi}_{m_t}|m_s)_\ell\, {}_Q(s_Q|, \tag{17}$$

with the help of Clebsch-Gordan coefficients $(\ell_1, m_1, \ell_2, m_2|\ell, m)$. Here, we first combine the orbital angular momentum and the isospin ($\vec{\lambda} = \vec{L} + \vec{I}_h$) and then combine the light quark spin. For a given $k_\ell(\neq 0)$, we have four $\mathcal{K}^{(i)}_{k_\ell k_3 s_Q}$ depending on ℓ and $\lambda^{(i)}$. (See Table 1.)

In terms of these $\mathcal{K}^{(i)}_{k_\ell k_3 s_Q}$, the heavy meson wavefunction can be written as

$$\begin{aligned} H_n(\vec{r}) &= \sum_{i=1,2} h^{(i)}_{k_\ell}(r)\mathcal{K}^{(i)}_{k_\ell k_3 s_Q}, \quad \text{for } \pi = -(-1)^{k_\ell} \text{ states}, \\ H_n(\vec{r}) &= \sum_{i=3,4} h^{(i)}_{k_\ell}(r)\mathcal{K}^{(i)}_{k_\ell k_3 s_Q}, \quad \text{for } \pi = +(-1)^{k_\ell} \text{ states}, \end{aligned} \tag{18}$$

with the radial functions $h^{(i)}_{k_\ell}(r)$. Note that i=1,2 states and i=3,4 states are decoupled due to parity.

We assume that the heavy meson and the soliton are infinitely heavy, in which case the heavy meson is just sitting at the center of the soliton where the potential has the lowest value. That is, in the heavy mass limit, all the radial functions $h^{(i)}_{k_\ell}(r)$ can be approximated as

$$h^i_{k_\ell}(r) = \alpha_i f(r), \tag{19}$$

with a constant α_i and a function $f(r)$ which is strongly peaked at the origin and normalized as $\int_0^\infty r^2 dr|f(r)|^2 = 1$. Then, the problem is reduced to solving the secular equation

$$\sum_j \mathcal{M}_{ij}\alpha_j = -\varepsilon\alpha_i, \qquad (i,j\text{=1,2 or 3,4}) \tag{20}$$

where the matrix elements $\mathcal{M}_{ij}(i,j$=1,2 or 3,4) are defined as

$$\mathcal{M}_{ij} = -\tfrac{1}{2}gF'(0)\int d\Omega\, \mathrm{Tr}\left\{\bar{\mathcal{K}}^{(i)}_{k_\ell k_3 s_Q}(\vec{\tau}\cdot\hat{r})[(\vec{\sigma}\cdot\vec{\tau})](\vec{\tau}\cdot\hat{r})\mathcal{K}^{(j)}_{k_\ell k_3 s_Q}\right\}, \tag{21}$$

with $\bar{\mathcal{K}} = \gamma^0\mathcal{K}^\dagger\gamma^0$. The minus sign of the energy in eq. (20) comes from the normalization of the basis states $\mathcal{K}$. We have used that $F(r) = \pi + F'(0)r + O(r^3)$ near the origin so that $a_1(r) \sim -F'(0) + O(r^2)$ and $a_2(r) \sim 2F'(0) + O(r^2)$ and the identity $(2\vec{\sigma}\cdot\hat{r}\vec{\tau}\cdot\hat{r} - \vec{\sigma}\cdot\vec{\tau}) = (\vec{\tau}\cdot\hat{r})(\vec{\sigma}\cdot\vec{\tau})(\vec{\tau}\cdot\hat{r})$.

For a given set of quantum numbers for k_ℓ, k_3 and s_Q, we have four eigenstates[#2]:

$$\begin{aligned}
\varepsilon &= -\tfrac{1}{2}gF'(0); \quad \mathcal{K}^{(1)}_{k_\ell k_3 s_Q}, \mathcal{K}^{(2)}_{k_\ell k_3 s_Q}, \mathcal{K}^{(+)}_{k_\ell k_3 s_Q} (= \sqrt{\tfrac{k_\ell}{2k_\ell+1}}\mathcal{K}^{(3)} + \sqrt{\tfrac{k_\ell+1}{2k_\ell+1}}\mathcal{K}^{(4)}), \\
\varepsilon &= +\tfrac{3}{2}gF'(0); \quad \mathcal{K}^{(-)}_{k_\ell k_3 s_Q} (= \sqrt{\tfrac{k_\ell+1}{2k_\ell+1}}\mathcal{K}^{(3)} - \sqrt{\tfrac{k_\ell}{2k_\ell+1}}\mathcal{K}^{(4)}).
\end{aligned} \tag{22}$$

Since $gF'(0) > 0$ (in case of baryon-number-one soliton solution), we have *three* bound states of the heavy mesons carrying antiflavor (C=-1 or B=$+1$) with a binding energy $\frac{1}{2}gF'(0)$. The unbound state with eigenenergy $+\frac{3}{2}gF'(0)$ corresponds to the bound state of the heavy meson with $C = +1$ or $B = -1$, whose eigenstates are given by negative energy eigenstates in our approach. It is interesting to note that a k_ℓ leads to two grand spins; *i.e.*, $k = k_\ell \pm \frac{1}{2}$ (unless $k_\ell = 0$). Thus, if one works with the grand spin $\vec{K}$ (instead of $\vec{K}_\ell$) along with the traditional bound state approach, the heavy quark spin symmetry implies that the eigenstates come in by degenerate doublets with grand spin $k = k_\ell \pm \frac{1}{2}$. When the heavy mesons have finite masses, these degeneracies should be removed. In other words, as the heavy meson masses increase, their eigenstates approach each other and become degenerate in the infinite heavy meson mass limit. It plays a nontrivial role in the quantization procedure.

In case of a typical soliton solution stabilized by the Skyrme term, when parameters are fixed as f_π=64.5 MeV and e=5.45 for the soliton to fit the nucleon and Delta masses [17], $F'(0)$ amounts to ~ -0.70 GeV. With $g = -0.75$ that the nonrelativistic quark model predicts, the binding energy of the normal heavy mesons and anti-flavored heavy mesons to the soliton are estimated as $\frac{3}{2}gF'(0) \sim 0.79$ GeV and $\frac{1}{2}gF'(0) \sim 0.26$ GeV, respectively. Comparing it with that of ref. [7], one can see that the binding energy for the bound antiflavored heavy meson is reduced by a factor of one half and more. It should be mentioned further that in ref. [7] the binding energy increases as the heavy meson masses and also that our results are obtained with infinite heavy meson masses.

However, the degeneracy in k_ℓ is an artifact originated from the approximation (19) on the radial function $h^{(i)}_{k_\ell}(r)$. In general, when the heavy meson's kinetic term is taken into account, the radial function feels the centrifugal potential $\ell_{\rm eff}(\ell_{\rm eff}+1)/r^2$ near the origin so that it behaves as $h^{(i)}_{k_\ell} \sim r^{\ell_{\rm eff}}$. Here, $\ell_{\rm eff}$ is the "effective" angular momentum [8], which is related with ℓ as

$$\ell_{\rm eff} = \begin{cases} \ell + 1, & \text{if } \lambda_i = \ell + \frac{1}{2}, \\ \ell - 1, & \text{if } \lambda_i = \ell - \frac{1}{2}. \end{cases} \tag{23}$$

Due to the vector potential $\vec{V}$ ($\sim i(\hat{r} \times \vec{\tau})/r$, near the origin) the singular structure of the *covariant* derivative squared $\vec{D}^2 = (\vec{\nabla} + \vec{V})^2$ is altered to $\ell_{\rm eff}(\ell_{\rm eff}+1)/r^2$ from the usual form of $\ell(\ell+1)/r^2$, which results from $\vec{\nabla}^2$. Thus, only those states with $\ell_{\rm eff} = 0$ can have strongly peaked radial function and the degeneracies will split such that the states with higher $\ell_{\rm eff}$ have higher energy. Note that $\ell_{\rm eff} = 0$ can be achieved only when $\ell = 1$.

It should be mentioned here that the wavefunctions appear in an ill-defined form at the

[#2] In case of $k_\ell = 0$, we have two eigenstates;

$$\begin{aligned}
\varepsilon &= -\tfrac{1}{2}gF'(0); \quad \mathcal{K}^{(1)}_{00 s_Q}(\hat{r}), \\
\varepsilon &= +\tfrac{3}{2}gF'(0); \quad \mathcal{K}^{(3)}_{00 s_Q}(\hat{r}).
\end{aligned}$$

origin[#3]. For example, the grand spin eigenfunction $K^{(2)}_{k_\ell=1,k_3,s_Q}$ is explicitly

$$
\begin{aligned}
K^{(2)}_{1,+1,s_Q} &= (\vec{\tau}\cdot\hat{r})\tilde{\phi}_{+\frac{1}{2}}|+\tfrac{1}{2})_{\ell\,Q}(s_Q|,\\
K^{(2)}_{1,0,s_Q} &= \tfrac{1}{\sqrt{2}}(\vec{\tau}\cdot\hat{r})\left\{\tilde{\phi}_{+\frac{1}{2}}|-\tfrac{1}{2})_{\ell\,Q}(s_Q| + \tilde{\phi}_{-\frac{1}{2}}|+\tfrac{1}{2})_{\ell\,Q}(s_Q|\right\},\\
K^{(2)}_{1,-1,s_Q} &= (\vec{\tau}\cdot\hat{r})\tilde{\phi}_{+\frac{1}{2}}|+\tfrac{1}{2})_{\ell\,Q}(s_Q|.
\end{aligned}
\tag{24}
$$

Since the radial function $h(r)$ does not vanish at the origin; *i.e.*, $r\sim r^{\ell_{\rm eff}}$ with $\ell_{\rm eff}=0$, $(\vec{\tau}\cdot\hat{r})$ in $K^{(2)}_{1k_3s_Q}$ leads an ill-defined wavefunction. Together with $\xi=\exp(i\vec{\tau}\cdot\hat{r}F(r)/2)$ which is also ill-defined at the origin, such ill-defined wavefunctions do not cause any problem in evaluating the physical quantities. It is entirely due to our convention for the representation of chiral symmetry. We have adopted, so-called the ξ-basis which has a simple transformation rule for the parity, while one would have well-defined wavefunctions in the Σ basis. (See ref. [18] for further details.)

To endow correct quantum numbers such as spin and isospin to the soliton-heavy meson bound system, we quantize the zero modes associated with the invariance under simultaneous $SU(2)$ rotation of the soliton configuration together with the heavy meson fields. We introduce the $SU(2)$ collective variables $C(t)$ as

$$
\xi(\vec{r},t) = C(t)\xi_0(\vec{r})C^\dagger(t), \qquad \text{and} \qquad H(\vec{r},t) = C(t)H_{\rm bf}(\vec{r},t). \tag{25}
$$

Here, $H_{\rm bf}$ refers to the heavy meson field in the body-fixed frame, while $H(\vec{r},t)$ refers to that in the laboratory frame. Substitution of eq. (25) into eq. (11) leads us to the lagrangian (in the reference frame where the heavy meson is at rest in space but rotating in isospin space)

$$
\begin{aligned}
L^{\rm rot} = -M_{sol} &+ \int d^3r\left\{-i\,{\rm Tr}(\bar{H}_{\rm bf}\partial_0 H_{\rm bf}) + g\,{\rm Tr}(\bar{H}_{\rm bf}\vec{A}\cdot\vec{\sigma}H_{\rm bf})\right\}\\
&+\tfrac{1}{2}\mathcal{I}\omega^2 + \int d^3r\tfrac{1}{2}\,{\rm Tr}\left\{\bar{H}_{\rm bf}\tfrac{1}{2}(\xi^\dagger\vec{\tau}\cdot\vec{\omega}\xi + \xi\vec{\tau}\cdot\vec{\omega}\xi^\dagger)H_{\rm bf}\right\},
\end{aligned}
\tag{26}
$$

where we have kept terms up to $O(m_Q^0 N_c^{-1})$. The "angular velocity", $\vec{\omega}$, of the collective rotation is defined by $C^\dagger\partial_0 C \equiv \frac{1}{2}i\vec{\tau}\cdot\vec{\omega}$.

The lagrangian (26) leads us to the Hamiltonian as

$$
\tilde{H} = M_{sol} - g\int d^3r\ {\rm Tr}(\bar{H}_{\rm bf}\,\vec{A}\cdot\vec{\sigma}\,H_{\rm bf}) + \frac{1}{2\mathcal{I}}(\vec{R}-\vec{\Theta}(\infty))^2, \tag{27}
$$

where the rotor spin $\vec{R}$ is canonical conjugate to the collective variables $C(t)$:

$$
R_a \equiv \frac{\delta L^{\rm rot}}{\delta\omega^a} = \mathcal{I}\omega_a + \Theta_a(\infty), \tag{27a}
$$

with $\vec{\Theta}(\infty)$ defined as

$$
\vec{\Theta}(\infty) \equiv +\tfrac{1}{2}\int d^3r\,{\rm Tr}\left\{\bar{H}_{\rm bf}\,\tfrac{1}{2}(\xi^\dagger\vec{\tau}\xi + \xi\vec{\tau}\xi^\dagger)\,H_{\rm bf}\right\}. \tag{27b}
$$

With the collective variable introduced as in eq. (25), the isospin of the fields $U(x)$ and $H(x)$ is entirely attributed to $C(t)$ and the isospin operator can be written in terms of the rotor spin as

$$
I_a = \tfrac{1}{2}\,{\rm Tr}(\tau_a C\tau_b C^\dagger)(\mathcal{I}\omega_b + \Theta_b(\infty)) = D^{\rm adj}_{ab}(C)R_b, \tag{28}
$$

[#3]We thank to the referee for pointing out this matter.

with $D^{\rm adj}_{ab}(C)$ being the $SU(2)$ adjoint representation associated with the collective variables $C(t)$. Furthermore, with the help of the K-symmetry in the solution, one can easily show that the spin of the $H_{\rm bf}$ is the grand spin; that is, the isospin of the H-field is transmuted into the part of the spin in the body-fixed frame. The spin of the soliton-heavy meson bound system is obtained as

$$\vec{J} = \vec{R} + \vec{K}^{\rm bf}, \tag{29}$$

with the grand spin $\vec{K}^{\rm bf}$ of the heavy meson fields in the body-fixed frame. Finally, the heavy-quark spin symmetry of the lagrangian under the transformation $H(x) \to H(x)S^{-1} = C(t)(H_{\rm bf}(x)S^{-1})$ has nothing to do with the collective rotations. Because of this heavy-quark spin decoupling, it is convenient to proceed with the spin operator $\vec{J}_\ell$ for the light degrees of freedom in the soliton-heavy meson bound system defined as

$$\vec{J}_\ell = \vec{J} - \vec{S}_Q = \vec{R} + \vec{K}^{\rm bf}_\ell. \tag{30}$$

Upon canonical quantization, the collective variables become the quantum mechanical operators; the isospin ($\vec{I}$), the spin ($\vec{J}_\ell$) and the rotor spin ($\vec{R}$) discussed so far become the corresponding operators $\tilde{I}_a$, $\tilde{J}_{\ell,a}$ and $\tilde{R}_a$, respectively. We distinguish those operators associated with the collective coordinate quantization by using a tilde on them. Let's denote the eigenstates of the rotor-spin operator $\tilde{R}_a$ as $|i; m_1, m_2\}$ ($m_1, m_2 = -i, -i+1, \cdots, i$):

$$\begin{aligned} \tilde{R}^2|i;m_1,m_2\} &= i(i+1)|i;m_1,m_2\}, \\ \tilde{R}_3|i;m_1,m_2\} &= m_2|i;m_1,m_2\}, \\ \tilde{I}_3|i;m_1,m_2\} &= m_1|i;m_1,m_2\}. \end{aligned} \tag{31}$$

Then, the eigenstates $|i, i_3; j_\ell, j_{\ell,3}; s_Q\rangle\!\rangle$ of the operators $\tilde{I}_a$ and $\tilde{J}_{\ell,a}$ with their corresponding quantum numbers i, i_3 (isospin) and $j_\ell, j_{\ell,3}$ (spin of the light degrees of freedom) are given by the linear combinations of the direct product of the rotor spin eigenstate $|i; m_1, m_2\}$ and the single particle Fock state $|n\rangle$:

$$|i, i_3; j_\ell, j_{\ell,3}; s_Q\rangle\!\rangle_a = \sum_m (i, j_{\ell,3} - m, k^a_\ell, m | j_\ell, j_{\ell,3}) |i; i_3, j_{\ell,3} - m\} |k_\ell, m, s_Q\rangle_a. \tag{32}$$

One may combine further the heavy quark spin and the spin of the light degrees of freedom to construct the states with a good total spin, which is not necessary however. Note that, in the infinite heavy quark mass limit, $(j_\ell, j_{\ell,3})$ themselves are good quantum numbers of the heavy hadrons together with the heavy quark spin due to the heavy quark symmetry. For a given set of (i, j^π_ℓ), there can be more than one state depending on which Fock state $|n\rangle$ is involved in the combination (32). We will distinguish them by using a sequential number, a(=1,2$\cdots$); *e.g.*, $|i, j^\pi_\ell\rangle\!\rangle_1$. Here again, to shorten the expressions, we will not specify such quantum numbers as i_3, j_3 and s_Q unless necessary. In Table 2, we list a few $|i, j^\pi_\ell\rangle\!\rangle$ states resulting from the soliton-antiflavored heavy meson bound system. Here, we have included only the integer rotor spin states so that the combined states can have a half-integer spin ($j = j_\ell \pm \frac{1}{2}$).

The physical P-baryons under consideration appear as the eigenstates of the Hamiltonian $\tilde{H}$. In case that we have only a single state $|i, j_\ell\rangle\!\rangle$ for a given quantum numbers (i, j^π_ℓ), it is the mass eigenstate and then the mass (modulo the heavy meson masses) of the corresponding baryon is simply obtained by evaluating the expectation value of the Hamiltonian with respect to the state:

$$M_{(i,j_\ell)} = M_{sol} + \varepsilon_n + \frac{1}{2\mathcal{I}}\{(1-c)i(i+1) + c j_\ell(j_\ell+1) - c k_\ell(k_\ell+1) + \tfrac{3}{4}\}, \tag{33}$$

Table 2 : $|i, j_\ell^\pi\rangle\!\rangle$ states for the P-baryons.

i	j_ℓ^π	$\lvert n\rangle$	$\lvert i, j_\ell^\pi\rangle\!\rangle_i$	ε	j
0	0^-	$\lvert 0\rangle_1$	$\lvert 0, 0^-\rangle\!\rangle$	$-\frac{1}{2}gF'(0)$	$\frac{1}{2}$
0	1^-	$\lvert 1\rangle_+$	$\lvert 0, 1^-\rangle\!\rangle$	$-\frac{1}{2}gF'(0)$	$\frac{1}{2}, \frac{3}{2}$
1	1^-	$\lvert 0\rangle_1$ $\lvert 1\rangle_+$ $\lvert 2\rangle_1$ $\lvert 2\rangle_2$	$\lvert 1, 1^-\rangle\!\rangle_1$ $\lvert 1, 1^-\rangle\!\rangle_2$ $\lvert 1, 1^-\rangle\!\rangle_3$ $\lvert 1, 1^-\rangle\!\rangle_4$	$-\frac{1}{2}gF'(0)$	$\frac{1}{2}, \frac{3}{2}$
0	1^+	$\lvert 1\rangle_1$ $\lvert 1\rangle_2$	$\lvert 0, 1^+\rangle\!\rangle_1$ $\lvert 0, 1^+\rangle\!\rangle_2$	$-\frac{1}{2}gF'(0)$	$\frac{1}{2}, \frac{3}{2}$
1	0^+	$\lvert 1\rangle_1$ $\lvert 1\rangle_2$	$\lvert 1, 0^+\rangle\!\rangle_1$ $\lvert 1, 0^+\rangle\!\rangle_2$	$-\frac{1}{2}gF'(0)$	$\frac{1}{2}$
1	1^+	$\lvert 1\rangle_1$ $\lvert 1\rangle_2$ $\lvert 2\rangle_+$	$\lvert 1, 1^+\rangle\!\rangle_1$ $\lvert 1, 1^+\rangle\!\rangle_2$ $\lvert 1, 1^+\rangle\!\rangle_3$	$-\frac{1}{2}gF'(0)$	$\frac{1}{2}, \frac{3}{2}$

where ε_n and k_ℓ are the eigenenergy and the light-quark grand spin of the heavy meson bound state involved in the combination of $|i, j_\ell^\pi\rangle\!\rangle$. We have used that the expectation values of the operators $\vec{\Theta}(\infty)$ and $\vec{\Theta}^2(\infty)$ with respect to the same single particle Fock state $|n\rangle$ are

$$\langle n|\vec{\Theta}(\infty)|n\rangle = -c\langle n|\vec{K}_\ell|n\rangle, \tag{33a}$$

with a constant c, and for all $|n\rangle$

$$\langle n|\vec{\Theta}^2(\infty)|n\rangle = \tfrac{3}{4}. \tag{33b}$$

Note that the mass formula (33) respects the heavy quark symmetry, *regardless of the c-value*, and that eq. (33b) is different from what would have been obtained by using the approximation of the traditional bound state approaches [8]

$$\langle n|\vec{\Theta}^2(\infty)|n\rangle \approx |\langle n|\vec{\Theta}(\infty)|n\rangle|^2 = c^2 k_\ell(k_\ell + 1). \tag{34}$$

If we have more than one state, say $|i, j_\ell^\pi\rangle\!\rangle_a (a{=}1, 2, \cdots)$, the situation is little bit more complicated. In general, each state alone cannot be an eigenstate of the Hamiltonian. Since they are degenerate up to the order N_c^0, the mass cannot be simply approximated by the expectation value of the Hamiltonian with respect to each state. The mass and the corresponding eigenstate are obtained by diagonalizing the energy matrix $\mathcal{E}$ defined as

$$\mathcal{E}_{ab} = (M_{sol} + \varepsilon_n)\delta_{ab} + \frac{1}{2\mathcal{I}}{}_a\langle\!\langle i, j_\ell^\pi|(\vec{R} - \vec{\Theta}(\infty))^2|i, j_\ell^\pi\rangle\!\rangle_b. \quad (a, b = 1, 2, \cdots) \tag{35}$$

We proceed with the i=1, j_ℓ^π=1^+ state as an illustration. The Wigner-Eckart theorem enables us to write down the expectation value of $\vec{\Theta}$ with respect to the Fock state as

$${}_a\langle k_\ell', m'|\Theta^q(\infty)|k_\ell, m\rangle_b = \frac{(k_\ell k_3 1 q|k_\ell' k_3')}{\sqrt{2k_\ell' + 1}}\,{}_a(k_\ell'\|\vec{\Theta}\|k_\ell)_b \tag{36}$$

Table 3 : Positive and Negative Parity P-baryon masses (in MeV).

i	j_ℓ^π	j^π	Mass Formula	$m_{P_{\bar{c}}}$	$m_{P_{\bar{b}}}$	b.e.*
0	0^-	$\frac{1}{2}^-$	$M_{sol}+\overline{m}_\Phi-\frac{1}{2}gF'(0)+3/8\mathcal{I}$	2704	6042	210
0	1^-	$\frac{1}{2}^-,\frac{3}{2}^-$	$M_{sol}+\overline{m}_\Phi-\frac{1}{2}gF'(0)+3/8\mathcal{I}$	2704	6042	210
1	1^-	$\frac{1}{2}^-,\frac{3}{2}^-$	$M_{sol}+\overline{m}_\Phi-\frac{1}{2}gF'(0)+7/8\mathcal{I}$	2802	6140	112
0	1^+	$\frac{1}{2}^+,\frac{3}{2}^+$	$M_{sol}+\overline{m}_\Phi-\frac{1}{2}gF'(0)+3/8\mathcal{I}$	2704	6042	210
1	0^+	$\frac{1}{2}^+$	$M_{sol}+\overline{m}_\Phi-\frac{1}{2}gF'(0)+7/8\mathcal{I}$	2802	6140	112
1	1^+	$\frac{1}{2}^+,\frac{3}{2}^+$	$M_{sol}+\overline{m}_\Phi-\frac{1}{2}gF'(0)+7/8\mathcal{I}$	2802	6140	112

* binding energy below the nucleon-heavy meson threshold.

with the "reduced matrix element" $(k'_\ell\|\vec{\Theta}\|k_\ell)$ and $\Theta^{\pm1}\equiv\mp\frac{1}{\sqrt{2}}(\Theta_x\pm i\Theta_y)$, $\Theta^0\equiv\Theta_z$. With the help of eqs. (33b) and (36), we obtain the energy matrix with respect to the three states $|1,1^+\rangle\rangle_a (a=1,2,3)$[#4]

$$\mathcal{E}_{(1,1^+)}=M_{sol}-\tfrac{1}{2}gF'(0)+\frac{11}{8\mathcal{I}}+\frac{1}{4\mathcal{I}}\begin{pmatrix}-1 & 0 & \sqrt{3}\\ 0 & 2 & 0\\ \sqrt{3} & 0 & 1\end{pmatrix}. \tag{37}$$

It leads us to three mass eigenvalues as

$$\begin{array}{l} M^-_{(1,1^+)}=M_{sol}-\frac{1}{2}gF'(0)+7/8\mathcal{I},\\ M^+_{(1,1^+)}=M_{sol}-\frac{1}{2}gF'(0)+15/8\mathcal{I},\ \text{(doubly degenerate)}\end{array} \tag{38}$$

only the first of which is below the nucleon-heavy meson threshold. The rotational energies are sufficiently large to make the other states unbound. In Table 3, listed are mass formulas for the positive and negative parity P-baryon states obtained in a similar way. We present only the states expected to be below the threshold. The degenerate mass of the P-baryons with the same isospin and spin but different parity is very interesting. However, we are not in the position to conclude whether such a parity doubling has any physical importance or just an artifact from our approximation on the radial functions.

Though very rough, at this point, we may give a prediction on the P-baryon masses. To this purpose, we add the heavy meson masses $\overline{m}_\Phi(\equiv\frac{1}{4}(3m_{\Phi^*}+m_\Phi)$, the weight average of the heavy meson masses; $\overline{m}_D$=1975 MeV and $\overline{m}_B$=5313 MeV) to the mass formula. Next, we fit the parameters f_π, e and g (equivalently, M_{sol}, $1/\mathcal{I}$ and $gF'(0)$) so as to yield correct masses of the nucleon, Delta [17] and Λ_c [10]:

$$\begin{array}{l} m_N=M_{sol}+3/8\mathcal{I}=939\ \text{MeV},\\ m_\Delta=M_{sol}+15/8\mathcal{I}=1232\ \text{MeV},\\ m_{\Lambda_c}=M_{sol}+\overline{m}_D-\frac{3}{2}gF'(0)+3/8\mathcal{I}=2285\ \text{MeV},\end{array} \tag{39}$$

[#4] Actually, we have two more states with $i=1$, $j_\ell^\pi=1^+$ made of *unbound* heavy meson states $|0\rangle_3$ and $|2\rangle_-$ combined with $i=1$ rotor spin state. We do not include these states into the procedure, since they have large energy discrepancy with the other three and their inclusion affects the eigenenergy only in the next order in $1/N_c$.

which lead to

$$M_{sol} = 866 \text{ MeV}, \quad 1/\mathcal{I} = 195 \text{ MeV} \quad \text{and} \quad gF'(0) = 419 \text{ MeV}. \tag{40}$$

Combined with the slope of the profile function $F'(0) \sim -690$ MeV (in case of the Skyrme-term-stabilized soliton solution), eq. (40) implies that $g \approx -0.61$ which is not far from that of the non-relativistic quark model (-0.75) and the experimental estimation ($|g|^2 \lesssim 0.5$). This set of parameters yields a prediction on the Λ_b and on the average mass of the Σ_c-Σ_c^* multiplets, $\overline{m}_{\Sigma_c} (\equiv \frac{1}{3}(2m_{\Sigma_c^*} + m_{\Sigma_c}))$ as

$$\begin{aligned} m_{\Lambda_b} &= M_{sol} + \overline{m}_B - \tfrac{3}{2} gF'(0) + 3/8\mathcal{I} = 5623 \text{ MeV}, \\ \overline{m}_{\Sigma_c} &= M_{sol} + \overline{m}_D - \tfrac{3}{2} gF'(0) + 11/8\mathcal{I} = 2483 \text{ MeV}, \end{aligned} \tag{41}$$

which are comparable with the experimental value of the Λ_b mass 5641 MeV and Σ_c mass 2453 MeV [19]. The P-baryon masses given in Table 3 could be accepted within the same error range.

However, all states listed in Table 3 do not seem to survive under the finite heavy meson mass corrections. Recently, we have reported that such finite mass corrections reduce the binding energy by an amount from 25% (in case of bottomed baryons) to 35% (in case of charmed baryons) of their infinite mass limit, $\frac{3}{2} gF'(0)$ [20]. Note that 35% of $\frac{3}{2} gF'(0)$ exceeds the binding energy $\frac{1}{2} gF'(0)$ for the soliton-antiflavored heavy mesons. Such an ambiguity prohibits us from any decisive conclusion before the finite mass corrections are incorporated [21]. Nonetheless, the states with $i = 0$, $j_\ell^\pi = 0^-$ and $i = 0$, $j_\ell^\pi = 1^\pm$ reveal a strong possibility for the non-strange P-baryon(s) different from the quark models. It supports the work of Riska and Scoccola [7], while the binding energy and the mass formula are quite different from theirs.

Acknowledgement

The work of YO was supported by the National Science Council of ROC under grant #NSC83-0208-M002-017. BYP and DPM were supported in part by the Korea Science and Engineering Foundation through the Center for Theoretical Physics, SNU.

References

[1] H.J. Lipkin, in *The elementary structure of matter*, Proc. Les Houches Workshop (April 1987), eds. J.M. Richard *et. al.* (Springer, Berlin, 1988); Phys. Lett. **B195** (1987) 484.

[2] C. Gignoux, B. Silvestre-Brac and J.M. Richard, in *The elementary structure of matter*, Proc. Les Houches Workshop (April 1987), eds. J.M. Richard *et. al.* (Springer, Berlin, 1988); Phys. Lett. **B193** (1987) 323.

[3] R.L. Jaffe, Phys. Rev. Lett. **38** (1977) 195.

[4] G. Karl and P. Żenczykowski, Phys. Rev. **D36** (1987) 3520;
Similar work has been done for the H-dibaryons by J.L. Rosner, Phys. Rev. **D33** (1986) 2043.

[5] S. Fleck, C. Gignoux, J.M. Richard and B. Silvestre-Brac, Phys. Lett. **B220** (1989) 616.

[6] S. Zouzou and J.-M. Richard, Few-Body Sys. **16** (1994) 1.

[7] D.O. Riska and N.N. Scoccola, Phys. Lett. **B299** (1993) 338.

[8] C.G. Callan and I. Klebanov, Nucl. Phys. **B262** (1985) 365;
M. Rho, D.O. Riska and N.N. Scoccola, Phys. Lett. **B251** (1990) 597; Z. Phys. **A341** (1992) 343;
D.O. Riska and N.N. Scoccola, Phys. Lett. **B265** (1991) 188;
Y. Oh, D.-P. Min, M. Rho and N.N. Scoccola, Nucl. Phys. **A534** (1991) 493.

[9] N. Isgur and M.B. Wise, Phys. Lett. **B232** (1989) 113; Phys. Lett. **B237** (1990) 507;
For a review, see H. Georgi, in *Proc. of the Theoretical Advanced Study Institute* eds. R.K. Ellis *et. al.* (World Scientific, Singapore, 1992);
M.B. Wise, in *Proc. of the Sixth Lake Louise Winter Institute*, ed. B.A. Campbell (World Scientific, Singapore, 1991).

[10] E. Jenkins, A.V. Manohar and M.B. Wise, Nucl. Phys. **B396** (1993) 27;
E. Jenkins and A.V. Manohar, Phys. Lett. **B294** (1992) 273;
Z. Guralnik, M. Luke and A.V. Manohar, Nucl. Phys. **B390** (1993) 474;
M.A. Nowak, M. Rho and I. Zahed, Phys. Lett. **B303** (1993) 130;
H.K. Lee and M. Rho, Phys. Rev. **D48** (1993) 2329.

[11] M.B. Wise, Phys. Rev. **D45** (1992) 2118;
T.-M. Yan *et al.*, Phys. Rev. **D46** (1992) 1148;
A.F. Falk, B. Grinstein and M. Luke, Nucl. Phys. **B357** (1991) 185;
A.F. Falk and M. Luke, Phys. Lett. **B292** (1992) 119;
R. Casalbuoni *et al.*, Phys. Lett. **B292** (1992) 371; Phys. Lett. **B299** (1993) 139;
J. Schechter and A. Subbaraman, Phys. Rev. **D48** (1993) 332.

[12] T.-M. Yan *et. al.*, in ref. [11].

[13] S. Barlag *et. al.*, (ACCMOR Collaboration) Phys. Lett. **B278** (1992) 480.

[14] S. Burtler *et. al.*, (CLEO Collaboration) Phys. Rev. Lett. **69** (1992) 2041.

[15] P.J. Mulders, Phys. Rev. **D30** (1984) 1073;
B.-Y. Park, Z. Phys. **A331** (1988) 473.

[16] Y. Oh, D.-P. Min and B.-Y. Park, SNU preprint SNUTP-94/13 (1994).

[17] G.S. Adkins, C.R. Nappi and E. Witten, Nucl. Phys. **B228** (1983) 552.

[18] E. Jenkins, A.V. Manohar and M.B. Wise, in ref. [10].

[19] Particle Data Group, Phys. Rev. **D45** (1993) Part II.

[20] Y. Oh, B.-Y. Park and D.-P. Min, Phys. Rev. **D49** (1994) to be published.

[21] Y. Oh, B.-Y. Park and D.-P. Min, in preparation.